Abstract: Gaps, Almost disjoint families and a Ramsey ultrafilter

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Recall that a pair $(\mathcal{L}, \mathcal{R})$ is a pregap over ω if both $\mathcal{L}$ and $\mathcal{R}$ are families of infinite subsets of ω and $L \cap R =^* \emptyset$ for each $L \in \mathcal{L}$ and $R \in \mathcal{R}$. A pregap $(\mathcal{L}, \mathcal{R})$ is a gap if there is no infinite subset of ω, say C, so that $L \subseteq^* C$ and $R \cap C =^* \emptyset$ for any $L \in \mathcal{L}$ and $R \in \mathcal{R}$. Whenever X and Y are partial orders, we say that a pregap (resp. a gap) $(\mathcal{L}, \mathcal{R})$ is an (X, Y)-pregap (resp. an (X, Y)-gap) if X is order isomorphic to $(\mathcal{L}, \subseteq^*)$ and Y to $(\mathcal{R}, \subseteq^*)$. This talk is divided into three sections which we now describe:

- **Types of gaps:** (X, Y)-gaps have been widely studied throughout the history in the case where X and Y are both infinite cardinals. However, as far as I know, there are not so many ZFC results regarding the existence of gaps for arbitrary partial orders. In this first section we show that the structure of gaps over ω is as rich as it could be even without assuming extra axioms. Formally, we will prove that for any two partial orders X and Y of cofinality ω_1 there are cofinal $X' \subseteq X$ and $Y' \subseteq Y$ for which there is an (X', Y')-gap. This theorem will be proved by using a concept that we call "The Luzin representation of a partial order".
- **Donut-inseparable gaps** Restricting our attention to the case of (ω_1, ω_1)-gaps, we introduce the concept of strongly donut-separable gaps and weakly donut-inseparable gaps. Informally speaking, a gap $(\mathcal{L}, \mathcal{R})$ is weakly donut-inseparable if there is an almost disjoint family $\mathcal{A}$ (explicitly definable from the gap) testifying that $(\mathcal{L}, \mathcal{R})$ is in fact a gap. A gap is strongly donut-separable if it is not weakly donut-inseparable. In this second section we show that CH implies that any (ω_1, ω_1)-gap is weakly donut-inseparable. On the other hand, we introduce a new cardinal invariant which we call $\mathfrak{m}_{\mathcal{F}}$ and we show that $\mathfrak{m}_{\mathcal{F}} > \omega_1$ implies that there is a strongly donut-separable gap.
- **A Ramsey ultrafilter**. In this section we show that $\mathfrak{m}_{\mathcal{F}} > \omega_1$ implies the existence of a Ramsey (also called Selective) ultrafilter. A curious feature about this result is that this ultrafilter has an explicit definition using a recently studied structure over ω_1 (A construction scheme).

If there is any time left we will talk about how the Luzin representation of partial orders can be used to calculate the cardinality of the gap cohomology groups of some uncountable structures.